**Exam Principles of Measurement Systems (PoMS); WBPH029-05**
**Semester Ib (2022/2023)**
**23 January 2023 8:30-10:30 (+20 min, if applicable)**

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- This is a **closed book** exam, but you are allowed to use two A4 pages (one A4 sheet) of your own, prepared in advance, **hand-written** notes For calculations, a basic (non-graphical) calculator is allowed to be used
- **Do not use a pencil to write down your answers and calculations! Exams written down with a pencil will not be accepted! Always use a pen!**
- This exam contains **9 pages** (including this cover page) and **4 questions**
- The total amount of points is **90**
- **Always motivate your answers, clearly explain any assumptions**
- Round numerical answers to the correct amount of digits Show unit calculations
- Suspected fraud will be reported to the Board of Examiners

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1 (21 points) The following are short questions about the material of the course Select appropriate answers to the questions Multiple answers are possible

**1.1** (*1 point*) The goal of *repetition* is the following:

a. To compare measured data with a data set performed in a different laboratory

b To be competitive worldwide

c To double check the estimated value of a measure variable

d To improve the estimated value of a measured variable

**1.2** (*1 point*) Which statement about static or dynamic calibration is not correct?

a A variable of interest is static in a static calibration

b A variable of interest is dynamic in a dynamic calibration

c Input values in static calibration depend on time and space

d Input values in dynamic calibration depend on time and space

**1.3** (*1 point*) Accuracy refers to

a The closeness of agreement between the expected measured error and true error value

b The closeness of agreement between the measured value and the true value

c. The closeness of agreement between the associated uncertainties and the error value

d None of them

**1.4** (*1 point*) To calculate the overall instrument uncertainty all partial and identified instrument's uncertainties need to be considered. Indicate the correct step(s).

a They should be simply added

b. They should be added and squared

c A square root of each partial uncertainty should be taken and then all should be added and squared

d They should be squared, added and a square root of all partial uncertainties should be taken

**1.5** (*2 points*) For which type of a signal does the magnitude remain constant between samples?

a. Analog signal

b Discrete time signal

c Digital signal

d. All of them.

**1.6** (*2 points*) Which characteristics apply to the nondeterministic signal?

a It has no recognizable pattern of repetition

b It is random in nature

c It can be predicted before it occurs

d It cannot be described by statistical characteristics

**1.7** (*1 point*) A frequency signal allows one to choose a proper measurement system and interpret the output signal This is possible due to.

a. Using systematic errors

b Averaging the signal and its error

c Applying logic unit electronic modules

d Fourier transform

**1.8** (*2 point*) Which answer describes modes for vibrating strings?

a The amplitude increases with increased harmonic number

b. The amplitude decreases with increased harmonic number

c Modes for vibrating strings are described by a phase shift

d. A frequency decreases with decreased harmonic number

**1.9** (*1 point*) In a woodwind instruments (e.g a flute) the pitch/frequency goes down in the following situation.

a When the wavelength is shorter (effective length of the instrument is shorter)

b When the wavelength is longer (effective length of the instrument is larger)

c. The effective length of a woodwind instrument does not influence the obtained frequency

**1.10** (*1 point*) If there is a very complex system model to be analysed and simulated, how this can be approached?

a. By designing a simulation of a very complex (higher order) system model

b. A very complex system model should be simplified to lower order systems

c It is impossible to analyse a very complex system model

d None of the answers above are possible.

**1.11** (*2 point*) Does the system output of the first-order system give an immediate response to the input signal? Justify your answer (write down on a paper).

a Yes

b No

**1.12** (*1 point*) What is the damping ratio for a non-oscillatory transient response?

a $\zeta < 1$

b $\zeta = 1$

c $\zeta > 1$

d $\zeta > 0$

**1.13** (*2 points*) Select correct answers describing statistical or systematic error

a Statistical error does not vary with repeated measurements

b. Statistical error is the (unknown) difference between the retained and true value

c Systematic error varies with repeated measurements

d Systematic error does not vary with repeated measurements

**1.14** (*1 point*) Direct integration of the Probability Density Function (PDF≡p(x)) for a normal distribution covers a percentage of the area under p(x) Which percentage representation is correct?

a 68,26% for z1=3 and 95,45% for z1=2 and 99,73% for z1=1

b 95,45% for z1=3 and 68,26% for z1=2 and 99,73% for z1=1

c. 68,26% for z1=2 and 95,45% for z1=1 and 99,73% for z1=3

d 68,26% for z1=1 and 95,45% for z1=2 and 99,73% for z1=3

**1.15** (*2 points*) Regression analysis is described by the following:

a It establishes a functional relationship between dependent and independent variables

b It assumes that a variation found in a dependent (measured) variable follows a normal distribution about each fixed value of an independent variable

c. It assumes that a variation found in a dependent (measured) variable follows a Poisson distribution about each fixed value of an independent variable

d. None of the above is correct

2 (26 points) A common device used for measuring gas pressure is the pencil-type pressure gauge It consists of a piston attached to a spring inside a tube. Exposed to a gas with a certain pressure, the piston is pressed, compressing the spring, until the restoring force of the spring (together with the ambient air pressure) balances the pressure of the gas A calibrated scale is printed on a plastic ruler attached to the back of the piston, allowing the user to read of the pressure as it protrudes out of the tube. The pressure gauge is shown in figure 1

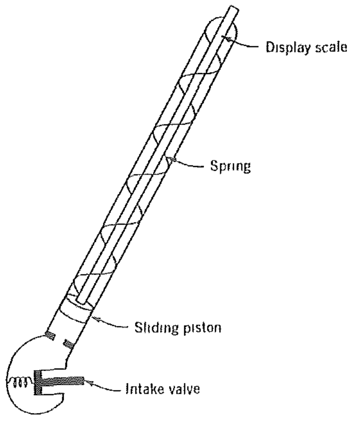


Figure 1. Schematic of a pencil-type pressure gauge The intake valve is exposed to the gas of which the pressure is to be measured This device is typically used for measuring air pressure in tyres

Assume the mass of the piston is not negligible and the system can be modeled by

$$my(t) + d\dot{y}(t) + ky(t) = F(t), \tag{1}$$

where $y(t)$ is the displacement of the piston and $F(t)$ is the force applied to the piston by the incoming gas The mass of the piston $m = 0\ 010$ kg, the damping constant $d = 1\ 2$ kg/s and the spring constant $k = 100$ N/m The surface area of the piston is 1 cm$^2$, 1 atm (= $10^5$ Pa) of pressure applies a force of 10 N on 1 cm$^2$ Assume the ambient pressure is 1 atm.

**a)** (*2 points*) What are practical advantages of having a strong spring (high $k$) and of having a weak spring (low $k$) in measuring various (constant) pressure values?

**b)** (*5 points*) Calculate the natural frequency of this system in rad/s and the damping ratio Include unit calculations Which system order (zero-, first- or second order) these calculations relate to and why?

**c)** (*9 points*) Calculate and then sketch the predicted output signal $y(t)$ for an incoming pressure of 2 atm (step change from 1 atm to 2 atm, $y(0) = 0$ m).

**d)** (*10 points*) Is the pressure gauge suitable (dynamic error $\leq 5\%$) for measuring a piston pressure force described by $F(t) = 80 + 40\sin(106t)$ N, based on its resonance frequency, magnitude ratio and phase shift? Explain/discuss why or why not.
(Assume the output can be digitized by some transducer, an oscillating analog scale would be difficult to read off manually after all )

3. (10 points) Experimental measurements are taken from a physical system that can be modeled as $y = a + 10\ln(x^n)$. The experimental data are (1.02, 40.32), (2.01, 33.65), (2.98, 29.52), (4.01, 26.27), (5.01, 24.12), (5.97, 22.12).

   **a)** (*10 points*) Find $a$ and $n$ using the method of least squares. (Hint: Transform the given model into a polynomial first)

4. (33 points) A pendulum oscillating in air will undergo damping due to friction. The amplitude, A, of oscillation will decrease exponentially as a function of time. That is $A(t)=A_0\exp(-\zeta t)$, where $\zeta$ is the damping coefficient. A student measures a pendulum's oscillation amplitude over time and from the collected data (see table 1) determines that $A_0 = 101.6$ mm and $\zeta = 0.884$. Make use of figure 2 and figure 3 printed on page 8 and 9.

| t (s) | A (mm) |
|---|---|
| 1 | 43.7 |
| 2 | 19.2 |
| 3 | 8.3 |
| 4 | 3.6 |
| 5 | 1.2 |
| 6 | 0.6 |
| 7 | 0.4 |
| 8 | 0.1 |

Table 1: Data collected of a damped pendulum's oscillation amplitude over time. There is an error of $\pm 0.1$ mm associated with all values of A.

   **a)** (*12 points*) Find the error in the fit, given the values for $A_0$ and $\zeta$. Explain performed steps.

   **b)** (*9 points*) Using the error found in part a), determine the uncertainty in the fit within a 95% confidence interval (if unable to do part a), use $s_{yx} = 0.5672$). Explain performed steps.

   **c)** (*12 points*) Find $\chi^2$ and from this determine if the fit is suitable to the acceptance level of 95%. If the fit is not suitable provide an explanation and suggest an improvement to the measurement.

# Formula sheet and useful tables

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$y(t) = A_0 + \sum_{n=1}^{\infty} \Big( A_n \cos(nt) + B_n \sin(nt) \Big)$$

$$A_0 = \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} y(t) dt$$

$$A_n = \frac{2}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} y(t) \cos(n\omega t) dt$$

$$B_n = \frac{2}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} y(t) \sin(n\omega t) dt$$

$$y(t) = KA + (y_0 - KA) e^{\frac{-t}{\tau}}$$

$$y(t) = Ce^{\frac{-t}{\tau}} + \frac{KA}{\sqrt{1 + (\omega\tau)^2}} \sin(\omega t - \tan^{-1}(\omega\tau))$$

$$M(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

$$\Phi(\omega) = -\tan^{-1}(\omega\tau)$$

$$\frac{1}{\omega_n^2} \ddot{y} + \frac{2\zeta}{\omega_n} \dot{y} + y = KF(t)$$

with $\omega_n = \sqrt{\frac{a_0}{a_2}}$, $\zeta = \frac{a_1}{2\sqrt{a_0 a_2}}$, $k = \frac{1}{a_0}$

$$\Phi(\omega) = \tan^{-}1 \left( -\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \right)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$M(\omega) = \frac{1}{\left( [1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2 \right)^{1/2}}$$

$$\omega_R = \omega_n \sqrt{1 - 2\zeta^2}$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$s_i^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$t = \frac{\bar{x} - x'}{s_x/\sqrt{N}}$$

$$z_0 = \frac{|x_i - \bar{x}|}{s_x}$$

$$\Delta y_{ci} = t_{\nu,P} \frac{s_{yx}}{\sqrt{N}}$$

$$s_{yx} = \sqrt{\frac{\sum_{i=1}^{N} (y_i - y_{ci})^2}{\nu}}$$

$$a = \frac{\Sigma x_i \Sigma x_i y_i - \Sigma x_i^2 \Sigma y_i}{(\Sigma x_i)^2 - N \Sigma x_i^2} \text{ for } y = a + bx$$

$$b = \frac{\Sigma x_i \Sigma y_i - N \Sigma x_i y_i}{(\Sigma x_i)^2 - N \Sigma x_i^2} \text{ for } y = a + bx$$

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{y_i - y_{ci}}{\Delta y_i} \right)^2$$

$\left(\frac{y_i - y_{ci}}{y_{ci}}\right)^2$

$$(1 - 2P(z_0)) < \frac{1}{2N}$$

$$y(t) = KA - KAe^{-\zeta\omega_n t} \left[ \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin\left(\omega_n t \sqrt{1 - \zeta^2}\right) + \cos\left(\omega_n t \sqrt{1 - \zeta^2}\right) \right]$$

Table 4.6 Values for $\chi^2_\alpha$

| $\nu$ | $\chi^2_{0.99}$ | $\chi^2_{0.975}$ | $\chi^2_{0.95}$ | $\chi^2_{0.90}$ | $\chi^2_{0.50}$ | $\chi^2_{0.05}$ | $\chi^2_{0.025}$ | $\chi^2_{0.01}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.000 | 0.000 | 0 000 | 0.016 | 0 455 | 3.84 | 5.02 | 6.63 |
| 2 | 0 020 | 0 051 | 0.103 | 0 211 | 1 39 | 5 99 | 7 38 | 9.21 |
| 3 | 0.115 | 0.216 | 0 352 | 0 584 | 2.37 | 7 81 | 9.35 | 11.3 |
| 4 | 0 297 | 0 484 | 0 711 | 1 06 | 3 36 | 9 49 | 11 1 | 13.3 |
| 5 | 0.554 | 0.831 | 1 15 | 1 61 | 4.35 | 11.1 | 12.8 | 15.1 |
| 6 | 0 872 | 1 24 | 1 64 | 2 20 | 5 35 | 12 6 | 14 4 | 16.8 |
| 7 | 1.24 | 1.69 | 2 17 | 2.83 | 6.35 | 14.1 | 16.0 | 18.5 |
| 8 | 1 65 | 2 18 | 2 73 | 3.49 | 7 34 | 15 5 | 17.5 | 20 1 |
| 9 | 2.09 | 2 70 | 3.33 | 4.17 | 8.34 | 16 9 | 19.0 | 21.7 |
| 10 | 2 56 | 3 25 | 3 94 | 4.78 | 9 34 | 18 3 | 20.5 | 23.2 |
| 11 | 3 05 | 3.82 | 4.57 | 5.58 | 10.3 | 19 7 | 21.9 | 24.7 |
| 12 | 3.57 | 4 40 | 5 23 | 6.30 | 11.3 | 21.0 | 23.3 | 26 2 |
| 13 | 4 11 | 5.01 | 5.89 | 7 04 | 12 3 | 22 4 | 24 7 | 27.7 |
| 14 | 4.66 | 5.63 | 6.57 | 7.79 | 13.3 | 23.7 | 26.1 | 29.1 |
| 15 | 5 23 | 6 26 | 7 26 | 8.55 | 14 3 | 25 0 | 27.5 | 30.6 |
| 16 | 5.81 | 6.91 | 7.96 | 9.31 | 15.3 | 26.3 | 28.8 | 32.0 |
| 17 | 6 41 | 7.56 | 8.67 | 10.1 | 16 3 | 27.6 | 30 2 | 33.4 |
| 18 | 7.01 | 8.23 | 9.39 | 10.9 | 17.3 | 28.9 | 31.5 | 34.8 |
| 19 | 7.63 | 8.91 | 10 1 | 11.7 | 18.3 | 30.1 | 32.9 | 36.2 |
| 20 | 8 26 | 9.59 | 10.9 | 12.4 | 19 3 | 31 4 | 34 2 | 37.6 |
| 30 | 15.0 | 16.8 | 18 5 | 20.6 | 29.3 | 43.8 | 47.0 | 50.9 |
| 60 | 37 5 | 40 5 | 43.2 | 46.5 | 59.3 | 79 1 | 83.3 | 88.4 |

Figure 2 Table of $\chi^2$ values at a given acceptance level and degrees of freedom Note that $\nu = N - (m+1)$.

| $\nu$ | $t_{50}$ | $t_{90}$ | $t_{95}$ | $t_{99}$ |
|---|---|---|---|---|
| 1 | 1 000 | 6 314 | 12 706 | 63 657 |
| 2 | 0 816 | 2 920 | 4 303 | 9,925 |
| 3 | 0 765 | 2 353 | 3 182 | 5 841 |
| 4 | 0,741 | 2 132 | 2 770 | 4 604 |
| 5 | 0 727 | 2 015 | 2 571 | 4 032 |
| 6 | 0 718 | 1,943 | 2,447 | 3,707 |
| 7 | 0 711 | 1 895 | 2 365 | 3 499 |
| 8 | 0 706 | 1 860 | 2 306 | 3 355 |
| 9 | 0 703 | 1 833 | 2 262 | 3 250 |
| 10 | 0 700 | 1 812 | 2 228 | 3 169 |
| 11 | 0 697 | 1 796 | 2,201 | 3 106 |
| 12 | 0 695 | 1 782 | 2 179 | 3 055 |
| 13 | 0 694 | 1 771 | 2 160 | 3 012 |
| 14 | 0 692 | 1 761 | 2 145 | 2 977 |
| 15 | 0,691 | 1 753 | 2 131 | 2 947 |
| 16 | 0 690 | 1 746 | 2 120 | 2 921 |
| 17 | 0,689 | 1 740 | 2 110 | 2 898 |
| 18 | 0 688 | 1 734 | 2,101 | 2 878 |
| 19 | 0 688 | 1 729 | 2 093 | 2 861 |
| 20 | 0,687 | 1 725 | 2 086 | 2 845 |
| 21 | 0,686 | 1 721 | 2 080 | 2 831 |
| 30 | 0,683 | 1 697 | 2 042 | 2 750 |
| 40 | 0,681 | 1 684 | 2 021 | 2 704 |
| 50 | 0 680 | 1,679 | 2,010 | 2,679 |
| 60 | 0 679 | 1 671 | 2 000 | 2 660 |
| $\infty$ | 0,674 | 1 645 | 1 960 | 2 576 |

Figure 3. **Table 4.4**: Two sided Student's $t$-distribution as a function of degrees of freedom